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The sfermion-mass spectrum of the MSSM at the one-loop level

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Abstract. The sfermion-mass spectrum of the minimal supersymmetric standard model is investigated at the one-loop level. An on-shell scheme has been specified for renormalization of the basic breaking parameters of the sfermionic sector. Owing to $SU(2)$-invariance, the soft-breaking mass parameters of the left-chiral scalar fermions of each isospin doublet are identical. Thus, one of the sfermion masses of each doublet can be expressed in terms of the other masses and receives a mass shift at the one-loop level with respect to the lowest-order value, which can be of $O(10\,\text{GeV})$. Both strong and electroweak contributions have been calculated for scalar quarks and leptons.

1 Introduction

The next generation of high energy colliders will permit the discovery of supersymmetric particles and accurate measurements of their properties [1]. From a precise determination of masses, cross sections and asymmetries the fundamental parameters of the underlying theory [2] can be reconstructed. This knowledge will provide insight into the supersymmetry breaking mechanism and its relation to grand unification.

The reconstruction of the basic SUSY-parameters from the experimental data requires reliable relations between physical observables and fundamental parameters. Higher-order corrections to tree-level relations have to be calculated and have to be taken into account when transforming from physical parameters to the fundamental ones.

In this paper, we derive the one-loop contributions to the sfermion-mass spectrum. An on-shell renormalization scheme is applied where all masses are treated as pole masses. Due to the $SU(2)$-symmetry, the soft-breaking parameters of the supersymmetric partners of the left-handed fermions are identical. Hence, in each generation of up- and down-type sfermions one sfermion mass is dependent on the remaining masses in that generation. Therefore, on the one-loop level the pole mass of that sfermion receives a shift with respect to its tree-level value. This shift has been calculated including the complete set of one-loop diagrams. As a by-product, counterterms for the soft-breaking parameters of the sfermionic sector are derived and are being implemented into the MSSM version of FeynArts [3]. Since these counterterms are specified for the basic breaking parameters, they are different from those of [4], where another way of renormalization has been performed that consists of introducing counterterms directly for the physical parameters, i.e. masses and mixing angles instead of the soft-breaking parameters.

Beginning with a review of the sfermion-mass matrix on Born level in Sect. 2, renormalization conditions are specified and explicit mass counterterms are calculated in Sect. 3. In Sect. 4, we present our numerical results.

2 The scalar-fermion sector at the Born level

In the MSSM, supersymmetry breaking is implemented by explicitly adding soft-breaking terms to the symmetric Lagrangian. The sfermion-mass terms of the Lagrangian, for a given species of sfermions $\tilde{f}$, can be written as the bilinear expression

$$\mathcal{L}_{\tilde{f}-\text{mass}} = -\left(\tilde{f}_L^+, \tilde{f}_R^+\right) \mathcal{M}_{\tilde{f}} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}, \qquad (1)$$

with $\mathcal{M}_{\tilde{f}}$ as the sfermion-mass matrix squared (see top of next page).
Here the quantities M_L^2, $M_{\tilde{f}_R}^2$ and A_f denote the soft-breaking parameters. In this paper we treat these parameter as real quantities. $\tan\beta = \frac{v_2}{v_1}$ denotes the ratio of the Higgs vacuum expectation values v_1 and v_2, and μ is the supersymmetric Higgs mass parameter. As abbreviations, $c_{2\beta} = \cos(2\beta)$ and $s_W = \sin\theta_W$ are used where θ_W is the weak mixing angle. The parameter κ is defined as $\kappa = \cot\beta$ for up-type squarks and $\kappa = \tan\beta$ for down-type squarks and electron-type sleptons. m_f is the mass of the fermion f, Q_f the electromagnetic charge, and T_f^3 the isospin of f.

As far as we do not consider right-handed neutrinos within the MSSM, the corresponding superpartners do not exist. Thus, for sneutrinos the sfermion-mass matrix is 1-dimensional, with only the left-handed entry of (2).

The mass matrix (2) can be diagonalized by a transformation of the $\tilde{f}_{L,R}$ fields with the help of a unitary matrix $\mathcal{U}_{\tilde{f}}$,

$$\mathcal{M}_{\tilde{f}} = \begin{pmatrix} m_f^2 + M_L^2 + M_Z^2 c_{2\beta}\left(T_f^3 - Q_f s_{\mathrm{W}}^2\right) & m_f\left(A_f - \mu^* \kappa\right) \\ m_f\left(A_f^* - \mu\kappa\right) & m_f^2 + M_{\tilde{f}_R}^2 + M_Z^2 c_{2\beta} Q_f s_{\mathrm{W}}^2 \end{pmatrix}, \tag{2}$$

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} = \mathcal{U}_{\tilde{f}} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}, \qquad \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix} = \mathcal{U}_{\tilde{f}}^+ \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}. \tag{3}$$

In our case of real parameters, $\mathcal{U}_{\tilde{f}}$ can be parameterized in terms of the mixing angle $\theta_{\tilde{f}}$,

$$\mathcal{U}_{\tilde{f}} = \begin{pmatrix} \cos\theta_{\tilde{f}} & \sin\theta_{\tilde{f}} \\ -\sin\theta_{\tilde{f}} & \cos\theta_{\tilde{f}} \end{pmatrix}. \tag{4}$$

In the $(1,2)$-basis, the squared-mass matrix is diagonal,

$$\mathcal{D}_{\tilde{f}} = \mathcal{U}_{\tilde{f}} \mathcal{M}_{\tilde{f}} \mathcal{U}_{\tilde{f}}^+ = \begin{pmatrix} m_{\tilde{f}_1}^2 & 0 \\ 0 & m_{\tilde{f}_2}^2 \end{pmatrix}, \tag{5}$$

with the eigenvalues $m_{\tilde{f}_1}^2$ and $m_{\tilde{f}_2}^2$ given by

$$\begin{aligned} m_{\tilde{f}_{1,2}}^2 = \frac{1}{2}\left(M_L^2 + M_{\tilde{f}_R}^2\right) + m_f^2 + \frac{1}{2} T_f^3 M_Z^2 c_{2\beta} \\ \pm \frac{1}{2}\left\{ \left[M_L^2 - M_{\tilde{f}_R}^2 + M_Z^2 c_{2\beta}\left(T_f^3 - 2Q_f s_{\mathrm{W}}^2\right)\right]^2 \right. \\ \left. + 4 m_f^2 |A_f - \mu\kappa|^2 \right\}^{1/2}. \end{aligned} \tag{6}$$

3 The scalar-fermion sector at the one-loop level

For renormalization of the sfermion sector, counterterms for the mass matrix (2) are introduced,

$$\mathcal{M}_{\tilde{f}} \to \mathcal{M}_{\tilde{f}} + \delta\mathcal{M}_{\tilde{f}}, \tag{7}$$

where $\delta\mathcal{M}_{\tilde{f}}$ contains the counterterms of the parameters appearing in (2).

By field renormalization, the sfermion fields are replaced by renormalized fields and Z-factors,

$$\begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix} \to \left(\mathbb{1} + \frac{1}{2}\delta\mathcal{Z}_{\tilde{f}}\right) \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix},$$

with

$$\delta\mathcal{Z}_{\tilde{f}} = \begin{pmatrix} \delta Z_{\tilde{f}L} & 0 \\ 0 & \delta Z_{\tilde{f}R} \end{pmatrix}. \tag{8}$$

This assignment forms the minimal set of renormalization constants satisfying the symmetry relations [5] and is sufficient to absorb all the divergencies.

The renormalization transformations (7) and (8), together with (3), yield the renormalized sfermion self-energies $\hat{\Sigma}_{\tilde{f}}$ from the unrenormalized ones, $\Sigma_{\tilde{f}}$, according to

$$\begin{aligned} \hat{\Sigma}_{\tilde{f}}\left(k^2\right) = \Sigma_{\tilde{f}}\left(k^2\right) + k^2 \delta\tilde{\mathcal{Z}}_{\tilde{f}} - \frac{1}{2}\left(\delta\tilde{\mathcal{Z}}_{\tilde{f}} \mathcal{D}_{\tilde{f}} + \mathcal{D}_{\tilde{f}} \delta\tilde{\mathcal{Z}}_{\tilde{f}}\right) \\ - \mathcal{U}_{\tilde{f}} \delta\mathcal{M}_{\tilde{f}} \mathcal{U}_{\tilde{f}}^+. \end{aligned} \tag{9}$$

$\Sigma_{\tilde{f}}$ denotes the matrix of the diagonal and non-diagonal self-energies for $\tilde{f}_{1,2}$. $\delta\tilde{\mathcal{Z}}_{\tilde{f}}$ is used as an abbreviation, $\delta\tilde{\mathcal{Z}}_{\tilde{f}} = \mathcal{U}_{\tilde{f}} \delta\mathcal{Z}_{\tilde{f}} \mathcal{U}_{\tilde{f}}^+$.

It is convenient to introduce, instead of (3), a more general transformation at the one-loop level, replacing

$$\mathcal{U}_{\tilde{f}} \to \mathcal{R}_{\tilde{f}} = \left(\mathbb{1} + \frac{1}{2}\delta\mathcal{Z}_{U_{\tilde{f}}}\right)\mathcal{U}_{\tilde{f}}, \tag{10}$$

with an additional UV-finite matrix $\delta\mathcal{Z}_{U_{\tilde{f}}}$. This procedure yields a non-diagonal Z-matrix for the sfermion fields with four independent entries. In that case the renormalized self-energies are given by

$$\begin{aligned} \hat{\Sigma}_{\tilde{f}}\left(k^2\right) = \Sigma_{\tilde{f}}\left(k^2\right) + \frac{1}{2} k^2 \left(\delta\breve{\mathcal{Z}}_{\tilde{f}}^+ + \delta\breve{\mathcal{Z}}_{\tilde{f}}\right) \\ - \frac{1}{2}\left(\delta\breve{\mathcal{Z}}_{\tilde{f}}^+ \mathcal{D}_{\tilde{f}} + \mathcal{D}_{\tilde{f}} \delta\breve{\mathcal{Z}}_{\tilde{f}}\right) - \mathcal{U}_{\tilde{f}} \delta\mathcal{M}_{\tilde{f}} \mathcal{U}_{\tilde{f}}^+, \end{aligned} \tag{11}$$

with

$$\delta\breve{\mathcal{Z}}_{\tilde{f}} = \mathcal{U}_{\tilde{f}} \delta\mathcal{Z}_{\tilde{f}} \mathcal{U}_{\tilde{f}}^+ - \delta\mathcal{Z}_{U_{\tilde{f}}} = \begin{pmatrix} \delta\breve{Z}_{\tilde{f}_{11}} & \delta\breve{Z}_{\tilde{f}_{12}} \\ \delta\breve{Z}_{\tilde{f}_{21}} & \delta\breve{Z}_{\tilde{f}_{22}} \end{pmatrix}. \tag{12}$$

This procedure, in analogy to the one for renormalization of the chargino and neutralino sector performed in [6], will be the basis of the forthcoming discussion.

3.1 Renormalization conditions

All independent parameters in the sfermion-mass matrix $\mathcal{M}_{\tilde{f}}$ in (2) are replaced by renormalized parameters and the corresponding counterterms, which form the counterterm matrix $\delta\mathcal{M}_{\tilde{f}}$. Only the counterterms of the soft-breaking parameters M_L^2, $M_{\tilde{f}_R}^2$ and A_f have to be determined within the sfermion sector; the others follow from the gauge, gaugino, Higgs and fermion sectors.

For one generation of squarks, neglecting mixing between generations, there exists one mass matrix for the u-type squarks and one for the d-type squarks. Because of $SU(2)$-invariance, the parameter $M_{L_{\tilde{q}}}$ is the same for the u- and the d-type squarks. Therefore, in one generation of squarks, there are five parameters $M_{L_{\tilde{q}}}^2$, $M_{\tilde{u}_R}^2$, $M_{\tilde{d}_R}^2$, A_u, A_d with counterterms to be determined within the sfermion sector. Hence, five renormalization conditions are required.

On-shell mass-renormalization conditions can be imposed on both mass eigenstates of either the u- or d-type sfermions. Here we choose the isospin "+" system, with the on-shell conditions expressed in terms of the diagonal entries of (11),

$$\mathrm{Re}\,\hat{\Sigma}_{\tilde{u}_{ii}}\left(m_{\tilde{u}_i}^2\right) = 0 \quad \text{with} \quad i = 1, 2. \tag{13}$$

For the $\tilde{d}$ system, the on-shell condition is imposed for the $\tilde{d}_2$-squarks,

$$\mathrm{Re}\,\hat{\Sigma}_{\tilde{d}_{22}}\left(m^2_{\tilde{d}_2}\right) = 0\,, \tag{14}$$

as long as $\tilde{d}_2 \neq \pm\tilde{d}_L$. According to (6), we have chosen the heavier squark to be $\tilde{d}_1$ and the lighter one to be $\tilde{d}_2$, hence mixing angles $\theta_{\tilde{f}} > |\frac{\pi}{4}|$ can occur in the matrix $\mathcal{U}_{\tilde{f}}$ in (4). In case of $\tilde{d}_2 = \pm\tilde{d}_L$, corresponding to a mixing angle $\theta_{\tilde{d}} = \mp\frac{\pi}{2}$, the self-energy $\hat{\Sigma}_{\tilde{d}_{22}}$ contains only the counterterm $\delta M^2_{L_{\tilde{q}}}$ which is already fixed by one of the conditions (13). The renormalization condition (14) has to be replaced in that case by

$$\mathrm{Re}\,\hat{\Sigma}_{\tilde{d}_{11}}\left(m^2_{\tilde{d}_1}\right) = 0. \tag{15}$$

The three mass-renormalization conditions determine essentially the counterterms for the diagonal mass parameters, M^2_L, $M^2_{\tilde{u}_R}$ and $M^2_{\tilde{d}_R}$. The non-diagonal counterterms δA_u and δA_d can be fixed by imposing

$$\mathrm{Re}\,\hat{\Sigma}_{\tilde{u}_{12}}\left(m^2_{\tilde{u}_1}\right) + \mathrm{Re}\,\hat{\Sigma}_{\tilde{u}_{12}}\left(m^2_{\tilde{u}_2}\right) = 0 \tag{16}$$

$$\mathrm{Re}\,\hat{\Sigma}_{\tilde{d}_{12}}\left(m^2_{\tilde{d}_1}\right) + \mathrm{Re}\,\hat{\Sigma}_{\tilde{d}_{12}}\left(m^2_{\tilde{d}_2}\right) = 0. \tag{17}$$

The diagonal Z-factors of the field-renormalization matrix (10) can be determined by the condition that the residues of the sfermion propagators are unity,

$$\mathrm{Re}\,\frac{\partial\hat{\Sigma}_{\tilde{f}_{ii}}\left(k^2\right)}{\partial k^2}\bigg|_{k^2=m^2_{\tilde{f}_i}} = 0 \qquad \text{for} \quad i=1,2 \quad \text{and} \quad f=u,d\,. \tag{18}$$

There are two more non-diagonal Z-factors of (10) for each, u- and d-type, sfermion pair at our disposal. They can be exploited to have zero mixing on each mass-shell. Imposing

$$\mathrm{Re}\,\hat{\Sigma}_{\tilde{f}_{12}}\left(m^2_{\tilde{f}_2}\right) = 0 \quad \text{for} \quad f=u,d\,, \tag{19}$$

yields, together with (16) and (17), diagonal self-energies for each on-shell momentum k^2. Yet, one Z-factor for each pair of sfermions remains undetermined. With the convenient choice

$$\delta\breve{\mathcal{Z}}_{\tilde{f}_{12}} = \delta\breve{\mathcal{Z}}_{\tilde{f}_{21}} \quad \text{for} \quad f=u,d \tag{20}$$

one obtains by solving (16)–(20)

$$\delta\breve{\mathcal{Z}}_{\tilde{f}_{ii}} = -\,\mathrm{Re}\,\frac{\partial\Sigma_{\tilde{f}_{ii}}\left(k^2\right)}{\partial k^2}\bigg|_{k^2=m^2_{\tilde{f}_i}} \qquad \text{for} \quad i=1,2 \quad \text{and} \quad f=u,d\,, \tag{21}$$

$$\delta\breve{\mathcal{Z}}_{\tilde{f}_{12}} = \delta\breve{\mathcal{Z}}_{\tilde{f}_{21}} = -\frac{\mathrm{Re}\,\Sigma_{\tilde{f}_{12}}\left(m^2_{\tilde{f}_1}\right) - \mathrm{Re}\,\Sigma_{\tilde{f}_{12}}\left(m^2_{\tilde{f}_2}\right)}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}} \qquad \text{for} \quad f=u,d\,. \tag{22}$$

For sleptons, the renormalization procedure can be applied analogously. Since we have not introduced right-handed neutrinos, only the counterterms for the soft-breaking parameters $M^2_{L_{\tilde{l}}}$, $M^2_{\tilde{e}_R}$ and A_e have to be determined. Choosing the conditions in analogy to (13), (14) and (17) we get

$$\mathrm{Re}\,\hat{\Sigma}_{\tilde{\nu}}\left(m^2_{\tilde{\nu}}\right) = 0\,, \quad \mathrm{Re}\,\hat{\Sigma}_{\tilde{e}_2}\left(m^2_{\tilde{e}_2}\right) = 0\,, \tag{23}$$

$$\mathrm{Re}\,\hat{\Sigma}_{\tilde{e}_{12}}\left(m^2_{\tilde{e}_1}\right) + \mathrm{Re}\,\hat{\Sigma}_{\tilde{e}_{12}}\left(m^2_{\tilde{e}_2}\right) = 0\,. \tag{24}$$

With the field and parameter renormalization constants determined in the way described above, the renormalization of the sfermion sector is completed. The counterterms are being implemented into the MSSM version of FeynArts [3] for completion at the one-loop level.

3.2 Determination of the renormalization constants

The diagonal entries of the matrix (11) of the renormalized self-energies, for on-shell values of k^2, are given by $(i=1,2)$

$$\begin{aligned}\hat{\Sigma}_{\tilde{f}\,ii}\left(m^2_{\tilde{f}_i}\right) &= \Sigma_{\tilde{f}\,ii}\left(m^2_{\tilde{f}_i}\right) - \left(\mathcal{U}_{\tilde{f}}\,\delta\mathcal{M}_{\tilde{f}}\,\mathcal{U}^+_{\tilde{f}}\right)_{ii}\\ &= \Sigma_{\tilde{f}\,ii}\left(m^2_{\tilde{f}_i}\right) - \delta m^2_{\tilde{f}_i}\,.\end{aligned} \tag{25}$$

Solving the set of (13) and (14) for the mass renormalization, three out of the four squark-mass counterterms are determined as follows:

$$\delta m^2_{\tilde{u}_1} = \mathrm{Re}\,\Sigma_{\tilde{u}_{11}}\left(m^2_{\tilde{u}_1}\right), \tag{26}$$

$$\delta m^2_{\tilde{u}_2} = \mathrm{Re}\,\Sigma_{\tilde{u}_{22}}\left(m^2_{\tilde{u}_2}\right), \tag{27}$$

$$\delta m^2_{\tilde{d}_2} = \mathrm{Re}\,\Sigma_{\tilde{d}_{22}}\left(m^2_{\tilde{d}_2}\right). \tag{28}$$

The fourth mass counterterm is no longer independent and can be expressed by the other counterterms of the soft-breaking parameters in the following way:

$$\begin{aligned}\delta m^2_{\tilde{d}_1} &= U^2_{\tilde{d}_{11}}\delta M^2_{L_{\tilde{q}}} + 2U_{\tilde{d}_{11}}U_{\tilde{d}_{12}}\delta A_d + U^2_{\tilde{d}_{12}}\delta M^2_{\tilde{d}_R}\\ &\quad + U^2_{\tilde{d}_{11}}\delta C_{\tilde{d}_{11}} + 2U_{\tilde{d}_{11}}U_{\tilde{d}_{12}}\delta C_{\tilde{d}_{12}} + U^2_{\tilde{d}_{12}}\delta C_{\tilde{d}_{22}}\,,\end{aligned} \tag{29}$$

with

$$\begin{aligned}\delta C_{\tilde{f}_{11}} &= 2m_f\delta m_f - Q_f M_Z^2\cos(2\beta)\,\delta\sin^2(\theta_{\mathrm{W}})\\ &\quad + \left(T^3_f - Q_f\sin^2(\theta_{\mathrm{W}})\right)\left(\cos(2\beta)\,\delta M_Z^2 + M_Z^2\,\delta\cos(2\beta)\right),\end{aligned} \tag{30}$$

$$\delta C_{\tilde{f}_{12}} = \delta C_{\tilde{f}_{21}} = \delta m_f\left(A_f - \mu\kappa\right) - m_f\kappa\,\delta\mu - m_f\mu\,\delta\kappa$$

$$\text{with} \quad \kappa = \begin{cases}\cot\beta & \text{for up-type squarks, } f=u\,,\\ \tan\beta & \text{for down-type squarks, } f=d\,,\end{cases} \tag{31}$$

$$\begin{aligned}\delta C_{\tilde{f}_{22}} &= 2m_f\delta m_f + Q_f\big(M_Z^2\cos(2\beta)\,\delta\sin^2(\theta_{\mathrm{W}})\\ &\quad + \sin^2(\theta_{\mathrm{W}})\cos(2\beta)\,\delta M_Z^2\\ &\quad + M_Z^2\sin^2(\theta_{\mathrm{W}})\,\delta\cos(2\beta)\big)\,.\end{aligned} \tag{32}$$

The counterterms $\delta M^2_{L_{\tilde q}}$, $\delta M^2_{\tilde u_R,\tilde d_R}$, $\delta A_{u,d}$ for the basic parameters of the mass matrix $\mathcal{M}_{\tilde f}$ in (2) can be obtained from (25) and the non-diagonal entry $\delta Y_{\tilde f_{12}} = \left(\mathcal{U}_{\tilde f}\delta\mathcal{M}_{\tilde f}\mathcal{U}^+_{\tilde f}\right)_{12}$ as follows:

$$\delta M^2_{L_{\tilde q}} = U^2_{\tilde u_{11}}\delta m^2_{\tilde u_1} + U^2_{\tilde u_{12}}\delta m^2_{\tilde u_2} - 2U_{\tilde u_{12}}U_{\tilde u_{22}}\delta Y_{\tilde u_{12}} - \delta C_{\tilde u_{11}}\,, \tag{33}$$

$$\delta M^2_{\tilde u_R} = U^2_{\tilde u_{12}}\delta m^2_{\tilde u_1} + U^2_{\tilde u_{11}}\delta m^2_{\tilde u_2} + 2U_{\tilde u_{12}}U_{\tilde u_{22}}\delta Y_{\tilde u_{12}} - \delta C_{\tilde u_{22}}\,, \tag{34}$$

$$\begin{aligned}\delta M^2_{\tilde d_R} = {} & \frac{U^2_{\tilde d_{11}} - U^2_{\tilde d_{12}}}{U^2_{\tilde d_{11}}}\delta m^2_{\tilde d_2} + 2\frac{U_{\tilde d_{12}}U_{\tilde d_{22}}}{U^2_{\tilde d_{11}}}\delta Y_{\tilde d_{12}} \\ & + \frac{U^2_{\tilde d_{12}}U^2_{\tilde u_{11}}}{U^2_{\tilde d_{11}}}\delta m^2_{\tilde u_1} + \frac{U^2_{\tilde d_{12}}U^2_{\tilde u_{12}}}{U^2_{\tilde d_{11}}}\delta m^2_{\tilde u_2} \\ & - 2\frac{U^2_{\tilde d_{12}}U_{\tilde u_{12}}U_{\tilde u_{22}}}{U^2_{\tilde d_{11}}}\delta Y_{\tilde u_{12}} - \delta C_{\tilde d_{22}} \\ & + \frac{U^2_{\tilde d_{12}}}{U^2_{\tilde d_{11}}}\left(\delta C_{\tilde d_{11}} - \delta C_{\tilde u_{11}}\right)\,,\end{aligned} \tag{35}$$

$$\begin{aligned}\delta A_u = {} & \frac{1}{m_u}\Big[U_{\tilde u_{11}}U_{\tilde u_{12}}\left(\delta m^2_{\tilde u_1} - \delta m^2_{\tilde u_2}\right) \\ & + \left(U_{\tilde u_{11}}U_{\tilde u_{22}} + U_{\tilde u_{12}}U_{\tilde u_{21}}\right)\delta Y_{\tilde u_{12}} - \delta C_{\tilde u_{12}}\Big],\end{aligned} \tag{36}$$

$$\begin{aligned}\delta A_d = {} & \frac{1}{m_d}\left[-\frac{U_{\tilde d_{12}}}{U_{\tilde d_{11}}}\delta m^2_{\tilde d_2} + \frac{U_{\tilde d_{22}}}{U_{\tilde d_{11}}}\delta Y_{\tilde d_{12}}\right. \\ & + \frac{U_{\tilde d_{12}}U^2_{\tilde u_{11}}}{U_{\tilde d_{11}}}\delta m^2_{\tilde u_1} + \frac{U_{\tilde d_{12}}U^2_{\tilde u_{12}}}{U_{\tilde d_{11}}}\delta m^2_{\tilde u_2} \\ & - 2\frac{U_{\tilde d_{12}}U_{\tilde u_{12}}U_{\tilde u_{22}}}{U_{\tilde d_{11}}}\delta Y_{\tilde u_{12}} - \delta C_{\tilde d_{12}} \\ & \left. + \frac{U_{\tilde d_{12}}}{U_{\tilde d_{11}}}\left(\delta C_{\tilde d_{11}} - \delta C_{\tilde u_{11}}\right)\right]\,.\end{aligned} \tag{37}$$

$\delta Y_{\tilde f_{12}}$ is determined with the help of (16), (17) and (22) to be

$$\delta Y_{\tilde f_{12}} = \frac{1}{2}\left(\mathrm{Re}\,\Sigma_{\tilde f_{12}}\left(m^2_{\tilde f_1}\right) + \mathrm{Re}\,\Sigma_{\tilde f_{12}}\left(m^2_{\tilde f_2}\right)\right) \quad \text{for} \quad f = u, d\,. \tag{38}$$

Inserting the expressions for $\delta M^2_{L_{\tilde q}}$, $\delta M^2_{\tilde d_R}$ and δA_d into (29), the mass counterterm $\delta m^2_{\tilde d_1}$ can be written as

$$\begin{aligned}\delta m^2_{\tilde d_1} = {} & -\frac{U^2_{\tilde d_{12}}}{U^2_{\tilde d_{11}}}\delta m^2_{\tilde d_2} + 2\frac{U_{\tilde d_{12}}U_{\tilde d_{22}}}{U^2_{\tilde d_{11}}}\delta Y_{\tilde d_{12}} + \frac{U^2_{\tilde u_{11}}}{U^2_{\tilde d_{11}}}\delta m^2_{\tilde u_1} \\ & + \frac{U^2_{\tilde u_{12}}}{U^2_{\tilde d_{11}}}\delta m^2_{\tilde u_2} - 2\frac{U_{\tilde u_{12}}U_{\tilde u_{22}}}{U^2_{\tilde d_{11}}}\delta Y_{\tilde u_{12}} \\ & + \frac{1}{U^2_{\tilde d_{11}}}\left(\delta C_{\tilde d_{11}} - \delta C_{\tilde u_{11}}\right)\,.\end{aligned} \tag{39}$$

This relation contains, besides those counterterms determined within the sfermion sector, also renormalization constants that have to be taken from other sectors: the fermion-mass counterterm δm_f, the gauge-boson mass counterterms $\delta M^2_{W,Z}$, and $\delta\tan\beta$. The renormalization of the electroweak mixing angle, $\delta\sin^2\theta_{\mathrm W}$, follows from the relation $\sin^2\theta_{\mathrm W} = 1 - \frac{M^2_W}{M^2_Z}$ (actually, in the combination of (39), δM^2_Z drops out). $\delta\mu$ can be obtained from renormalization in the chargino sector and is given explicitly in [6]; $\delta m^2_{\tilde d_1}$ is, however, independent of $\delta C_{\tilde f_{12}}$ and hence also independent of $\delta\mu$.

The counterterms δm_f, $\delta\tan\beta$ and $\delta M^2_{W,Z}$ are determined from the following conditions.

(i) On-shell renormalization of the fermion mass yields [7]

$$\delta m_f = \frac{1}{2}m_f\left(\mathrm{Re}\,\Sigma_{f_L}\left(m^2_f\right) + \mathrm{Re}\,\Sigma_{f_R}\left(m^2_f\right) + 2\,\mathrm{Re}\,\Sigma_{f_S}\left(m^2_f\right)\right) \tag{40}$$

in terms of the fermion self-energy

$$\Sigma_f(k) = \Sigma_{f_L}\left(k^2\right)\not{k}\mathbf{P_L} + \Sigma_{f_R}\left(k^2\right)\not{k}\mathbf{P_R} + m_f\Sigma_{f_S}\left(k^2\right)\,. \tag{41}$$

(ii) On-shell renormalization of the gauge-boson masses determines the mass counterterms

$$\delta M^2_V = \mathrm{Re}\,\Sigma_V\left(M^2_V\right) \quad \text{for} \quad V = W, Z\,, \tag{42}$$

in terms of the vector-boson self-energies $\Sigma_V(k^2)$.

(iii) Vanishing A^0–Z-mixing for an on-shell A^0-boson determines the counterterm of $\tan\beta$ according to [8]

$$\delta\tan\beta = \frac{1}{2M_Z\cos^2\beta}\,\mathrm{Im}\,\Sigma_{A^0Z}\left(M^2_A\right)\,. \tag{43}$$

Another option is to renormalize $\tan\beta$ in the $\overline{\mathrm{DR}}$-scheme [9] where only the UV-singular part of (43) is taken into account, which has the advantage of avoiding large finite contributions and providing a gauge invariant and process independent counterterm [10]. Comparing both renormalization schemes, the numerical results for the sfermion masses differ by at most $\mathcal{O}(10\,\mathrm{MeV})$. A potentially large finite part of (43) for large values of $\tan\beta$ is suppressed by a factor $\tan\beta/(1+\tan^2\beta)^2$ in the sfermion-mass counterterms, which keeps the result stable.

After this specification of all the renormalization constants the squark sector is completed at the one-loop level. Another way of renormalization, performed in [4], consists of introducing counterterms directly for the physical parameters, i.e. masses and mixing angles, instead of the soft-breaking parameters. In that case also the transformation matrix $\mathcal{U}_{\tilde f}$ in (4) has to be renormalized by the mixing angle counterterm, i.e. through $\theta_{\tilde f} \to \theta_{\tilde f} + \delta\theta_{\tilde f}$, whereas in our case $\mathcal{U}_{\tilde f}$ is not affected. Previous studies of the sfermion-mass spectrum [11] were done in the $\overline{\rm DR}$-scheme with running parameters whereby the MSSM parameter space is restricted by unification assumptions.

The treatment of one generation of sleptons is similar to the one of squarks. Two of the three slepton masses are fixed by on-shell conditions, and the third one is dependent on the other counterterms,

$$\begin{aligned}\delta m^2_{\tilde{e}_1} &= U^2_{\tilde{e}_{11}}\delta M^2_{L_{\tilde{l}}} + 2U_{\tilde{e}_{11}}U_{\tilde{e}_{12}}\delta A_e + U^2_{\tilde{e}_{12}}\delta M^2_{\tilde{e}_R} \\ &\quad + U^2_{\tilde{e}_{11}}\delta C_{\tilde{e}_{11}} + 2U_{\tilde{e}_{11}}U_{\tilde{e}_{12}}\delta C_{\tilde{e}_{12}} + U^2_{\tilde{e}_{12}}\delta C_{\tilde{e}_{22}} . \quad (44)\end{aligned}$$

The quantities $\delta M^2_{L_{\tilde{l}}}$, $\delta M^2_{\tilde{e}_R}$ and δA_e follow from (23) and (24) and are explicitly given by

$$\delta M^2_{L_{\tilde{l}}} = \delta m^2_{\tilde{\nu}} - \delta C_{\tilde{\nu}} , \quad (45)$$

$$\begin{aligned}\delta M^2_{\tilde{e}_R} &= \frac{U^2_{\tilde{e}_{11}} - U^2_{\tilde{e}_{12}}}{U^2_{\tilde{e}_{11}}}\delta m^2_{\tilde{e}_2} + 2\frac{U_{\tilde{e}_{12}}U_{\tilde{e}_{22}}}{U^2_{\tilde{e}_{11}}}\delta Y_{\tilde{e}_{12}} + \frac{U^2_{\tilde{e}_{12}}}{U^2_{\tilde{e}_{11}}}\delta m^2_{\tilde{\nu}} \\ &\quad - \delta C_{\tilde{e}_{22}} + \frac{U^2_{\tilde{e}_{12}}}{U^2_{\tilde{e}_{11}}}\left(\delta C_{\tilde{e}_{11}} - \delta C_{\tilde{\nu}}\right) , \quad (46)\end{aligned}$$

$$\begin{aligned}\delta A_e &= \frac{1}{m_e}\left[-\frac{U_{\tilde{e}_{12}}}{U_{\tilde{e}_{11}}}\delta m^2_{\tilde{e}_2} + \frac{U_{\tilde{e}_{22}}}{U_{\tilde{e}_{11}}}\delta Y_{\tilde{e}_{12}} + \frac{U_{\tilde{e}_{12}}}{U_{\tilde{e}_{11}}}\delta m^2_{\tilde{\nu}} \right. \\ &\quad \left. - \delta C_{\tilde{e}_{12}} + \frac{U_{\tilde{e}_{12}}}{U_{\tilde{e}_{11}}}\left(\delta C_{\tilde{e}_{11}} - \delta C_{\tilde{\nu}}\right) \right] , \quad (47)\end{aligned}$$

with

$$\delta Y_{\tilde{e}_{12}} = \frac{1}{2}\left(\mathrm{Re}\,\Sigma_{\tilde{e}_{12}}\left(m^2_{\tilde{e}_1}\right) + \mathrm{Re}\,\Sigma_{\tilde{e}_{12}}\left(m^2_{\tilde{e}_2}\right)\right) , \quad (48)$$

$$\delta C_{\tilde{\nu}} = \frac{1}{2}\left(\cos(2\beta)\delta M_Z^2 + M_Z^2 \delta\cos(2\beta)\right) , \quad (49)$$

$$\begin{aligned}\delta C_{\tilde{e}_{11}} &= 2m_e\delta m_e + M_Z^2\cos(2\beta)\delta\sin^2(\theta_{\rm W}) \quad (50) \\ &\quad -\left(\frac{1}{2} - \sin^2(\theta_{\rm W})\right)\left(\cos(2\beta)\delta M_Z^2 + M_Z^2\delta\cos(2\beta)\right) ,\end{aligned}$$

$$\begin{aligned}\delta C_{\tilde{e}_{12}} = \delta C_{\tilde{e}_{21}} &= \delta m_e\left(A_e - \mu\tan\beta\right) - m_e\tan\beta\,\delta\mu \\ &\quad - m_e\mu\,\delta\tan\beta , \quad (51)\end{aligned}$$

$$\begin{aligned}\delta C_{\tilde{e}_{22}} &= 2m_e\delta m_e - M_Z^2\cos(2\beta)\delta\sin^2(\theta_{\rm W}) \\ &\quad - \sin^2(\theta_{\rm W})\cos(2\beta)\delta M_Z^2 \\ &\quad - M_Z^2\sin^2(\theta_{\rm W})\delta\cos(2\beta) . \quad (52)\end{aligned}$$

These expressions complete the renormalization also in the slepton sector.

3.3 Mass corrections

The sfermion masses fixed via the on-shell conditions (13) and (14) for squarks and (23) for sleptons do not receive any corrections at one-loop order. The remaining mass, in each squark or slepton generation, is different at tree level and one-loop order. The counterterm (29) or (44), respectively, absorbs the divergence of the corresponding self-energy, but it leaves a finite contribution. The shifts $\Delta m^2_{\tilde{d}_1}$ and $\Delta m^2_{\tilde{e}_1}$ for the pole masses are given by

$$\Delta m^2_{\tilde{d}_1} = \delta m^2_{\tilde{d}_1} - \mathrm{Re}\,\Sigma_{\tilde{d}_{11}}(m^2_{\tilde{d}_1})$$

and

$$\Delta m^2_{\tilde{e}_1} = \delta m^2_{\tilde{e}_1} - \mathrm{Re}\,\Sigma_{\tilde{e}_{11}}\left(m^2_{\tilde{e}_1}\right) , \quad (54)$$

yielding one-loop masses according to

$$m^2_{\tilde{d}_1 1-\mathrm{Loop}} = m^2_{\tilde{d}_{1\mathrm{Born}}} + \Delta m^2_{\tilde{d}_1}$$

and

$$m^2_{\tilde{e}_1 1-\mathrm{Loop}} = m^2_{\tilde{e}_{1\mathrm{Born}}} + \Delta m^2_{\tilde{e}_1} , \quad (56)$$

where $m_{\tilde{d}_{1\mathrm{Born}}}$ and $m_{\tilde{e}_{1\mathrm{Born}}}$ are the masses in the Born approximation. In the self-energies $\Sigma_{\tilde{d}_{11}}$ and $\Sigma_{\tilde{e}_{11}}$, the masses can be taken as the lowest-order masses.

4 Numerical results and discussion

The self-energies were calculated with the help of the programs FeynArts, FormCalc and LoopTools [3,12], with the method of "constrained differential renormalization" [13] for regularization. This method is equivalent to the procedure of dimensional reduction [14].

In the following, we illustrate the effect of the one-loop contributions for specific examples in Figs. 1 to 3. Unless stated otherwise, the default values for the parameters listed in Table 1 are used.

The size of the mass shift for the three squark generations is displayed in Fig. 1, together with the correction to the $\tilde{\tau}_1$ mass as an example for the sleptons. Because of the presence of the fermion mass in the off-diagonal entry of (2), the dependence of the sfermion masses on $\tan\beta$ is strongest for the third generation. The mass shifts are nearly independent of $\tan\beta$ for all the particles. They are rather small (0.6 GeV) in the first two squark generations, but they are much larger in the third generation. The Born mass of the $\tilde{b}_1$-squark is enhanced significantly by up to 16 GeV (5%) at the one-loop level. The mass shift for sleptons is of electroweak origin only and is hence rather small, for the $\tilde{\tau}_1$-slepton only 0.2 GeV (or 0.1%).

The various contributions to the one-loop mass shift versus $\tan\beta$ are shown in Fig. 2, for the case of the $\tilde{b}_1$ squark: the Born mass and the one-loop mass, together with the individual parts from the strong and the electroweak interactions. The biggest shift originates from the strong interaction, i.e. by virtual squarks, gluinos, quarks and gluons, and amounts to approximately 17 GeV (5%) and 6.5 GeV (1.5%) for $M_{L_{\tilde{q}_3}} = 300$ GeV and $M_{L_{\tilde{q}_3}} = 500$ GeV, respectively.

The electroweak contribution can become also more sizable, as the example of the right part in Fig. 2 shows, with $M_{L_{\tilde{q}_3}} = 500$ GeV, where a shift of 2.3 GeV is observed. The electroweak contributions result from virtual sleptons, squarks, charginos, neutralinos and quarks, Higgs-, W-

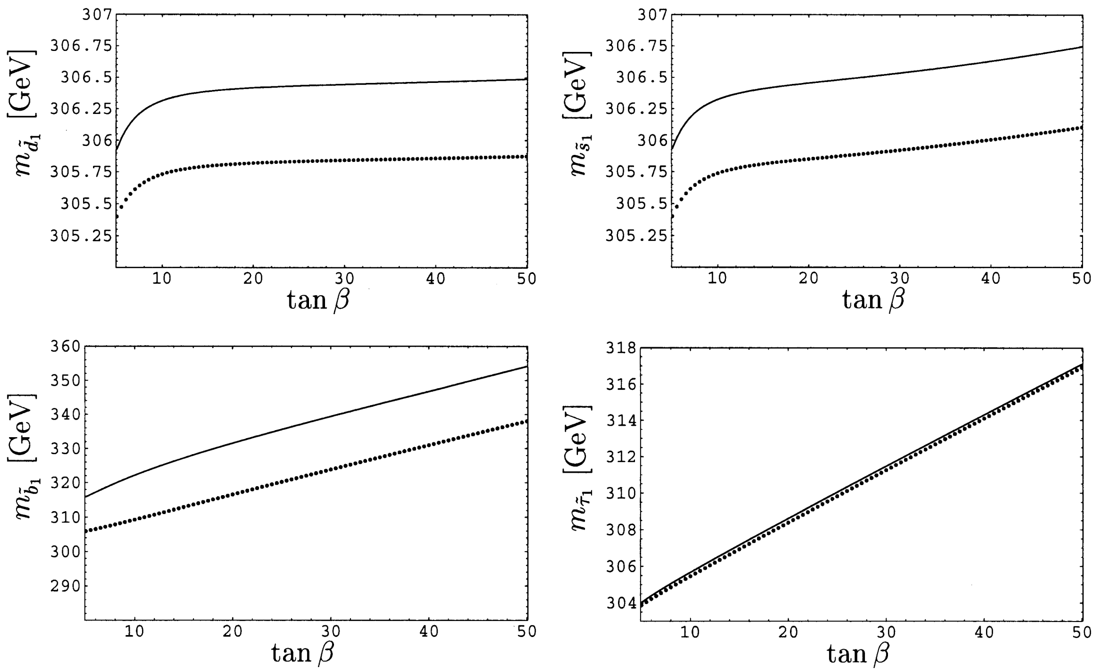

Fig. 1. Born (dotted line) and one-loop masses (solid line) as a function of $\tan\beta$. In the first row the masses of the $\tilde{d}_1$-squark and the $\tilde{s}_1$-squark are shown, in the second row the masses of the $\tilde{b}_1$-squark and the $\tilde{\tau}_1$-slepton. The parameters have been chosen as $M_{\tilde{f}_R} = M_L = A_f = 300\,\mathrm{GeV}$ for all generations

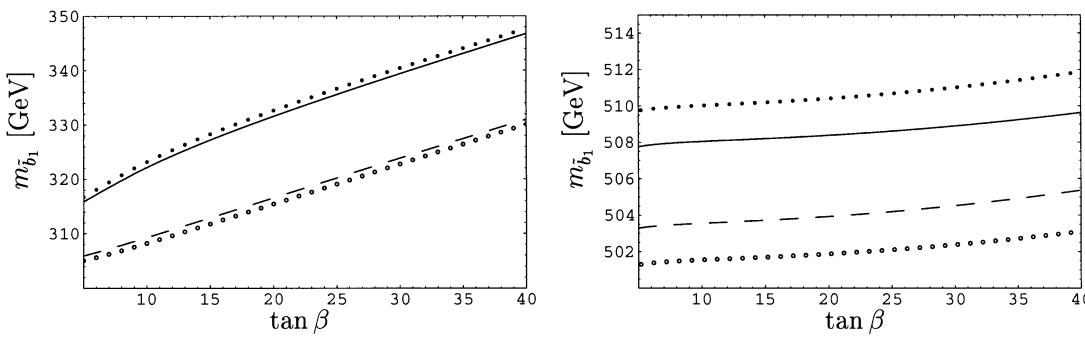

Fig. 2. Total one-loop mass (solid line) of the $\tilde{b}_1$-particle as a function of $\tan\beta$ in comparison to the Born mass (dashed line) and the one-loop masses with either strong (dotted line) and electroweak (line of circles) contributions. The soft-breaking parameters are chosen as $M_{L_{\tilde{q}_3}} = 300\,\mathrm{GeV}$ in the left and $M_{L_{\tilde{q}_3}} = 500\,\mathrm{GeV}$ in the right figure, and $M_{\tilde{f}_R} = M_{L_{\tilde{f}\neq\tilde{q}_3}} = A_f = 300\,\mathrm{GeV}$ in both

Table 1. If not mentioned explicitly in the text, the following default set of parameters is used

Parameters of the Higgs sector		
$M_A = 150\,\mathrm{GeV}$ (Mass of the A^0-boson)	$\tan\beta = 10$	$\mu = 100\,\mathrm{GeV}$
soft-breaking parameters		
for the gauginos:	for the sfermions:	
$M_1 = \dfrac{5}{3}\dfrac{\sin^2\theta_\mathrm{W}}{\cos^2\theta_\mathrm{W}} M_2$	$M_L = M_{L_{\{\tilde{q}_i,\tilde{l}_i\}}} = 300\,\mathrm{GeV}$ with $i = 1,2,3$	
$M_2 = 200\,\mathrm{GeV}$	$M_{\tilde{f}_R} = 300\,\mathrm{GeV}$ with $f = u,c,t,d,s,b,e,\mu,\tau$	
$M_3 = \dfrac{\alpha_\mathrm{s}}{\alpha}\sin^2\theta_\mathrm{W} M_2$	$A_{\{u,c,t\}} = A_{\{d,s,b\}} = A_{\{e,\mu,\tau\}} = 300\,\mathrm{GeV}$	

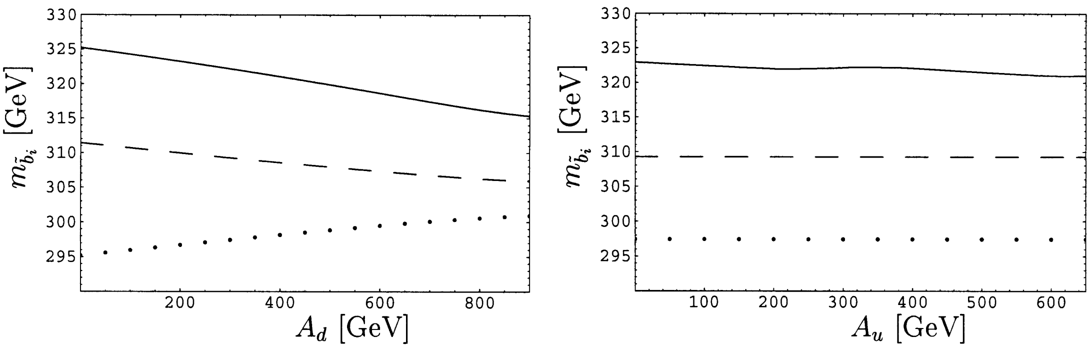

Fig. 3. Born (dashed line) and one-loop masses (solid line) of the $\tilde{b}_1$-squark and the mass of the $\tilde{b}_2$-squark (dotted line) as a function of the A-parameter A_d or A_u. The parameters have been chosen as $M_{\tilde{f}_R} = M_L = 300\,\text{GeV}$ for all generations. It is assumed that $A_d = A_s = A_b$ and $A_u = A_c = A_t$. When A_d is varied then $A_u = 300\,\text{GeV}$ and vice versa

and Z-bosons and photons. Since the strong and the electroweak contributions have opposite signs, the total correction adds up to 5% (16 GeV) and 1% (4.2 GeV) of the Born mass for $M_{L_{\tilde{q}_3}} = 300\,\text{GeV}$ and $M_{L_{\tilde{q}_3}} = 500\,\text{GeV}$, respectively.

Finally, the dependence on the A-parameters is considered. As an illustration, the A-parameter dependence is shown for the $\tilde{b}_i$-squarks in Fig. 3. Varying the parameter A_d changes the mixing of the bottom squarks, an effect which is suppressed by m_f in the light generations. The corrections to the Born mass show a weak dependence on the parameter A_d. They decrease from 14 GeV to 9 GeV in the A_d range of Fig. 3.

The Born masses of the bottom squarks do not depend on the parameter A_u, but one can see a slight decrease of the corrections to the mass of $\tilde{b}_1$ when A_u is increased. A_u changes the mixing and the mass splitting of the up-type squarks, which influences slightly the size of the mass shift.

5 Conclusion

We have presented a complete on-shell renormalization of the scalar-fermion sector of the MSSM based on the entire set of one-loop diagrams, treating all masses as pole masses and with renormalization constants that allow one to formulate the sfermion self-energies as matrices which become diagonal for external momenta on-shell. The renormalization conditions are specified to fix the counterterms of the basic soft-breaking parameters, respecting $SU(2)$-invariance. As an application, we have calculated the sfermion-mass spectrum at the one-loop level. Three of the four squark tree-level masses and two of the three slepton tree-level masses can be made equal to the corresponding one-loop pole masses. The residual squark and slepton mass, instead, receives a mass shift at one-loop level. These mass shifts are rather small for sleptons, but they can be sizable, of the order of 5%, for squarks. Thus, especially for the third generation, this mass shift has to be taken into account in precision calculations.

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